PB-003-001618 Seat No. ______

B. Sc. (Sem. VI) (CBCS) Examination

March / April - 2020

Mathematics : BSMT-603(A)

(Optimization & Numerical Analysis - II)
(Old Course)

Faculty Code : 003
Subject Code : 001618

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figure to the right indicate full marks of the question.

1 Give answers of all following questions : **20**

(1) Define Slack variable.

(2) Define Convex set.

(3) To solve maximization problem using Big-M method the coefficient for an artificial variable in objective function is ________.

(4) The method used to solve an assignment problem is called __________

(5) Full form of NWCM is ________

(6) In the optimal simplex table $z_j - c_j = 0$ indicates infeasible solution. (True/False)

(7) If for a given solution, a slack variable is equal to zero then the solution is optimal. (True/False)

(8) If there were n workers and n jobs, there would be $n\,!$ solutions. (True/False)

(9) While solving assignment problem an activity is assigned to resources through a square with zero opportunity cost because the objective is to reduce the cost of assignment to zero. (True/False)

(10) If an optimal solution is degenerate then the solution is infeasible. (True/False)

(11) Whenever Trapezoidal Rule is applicable, Simpson's 1/3 Rule can also be applied. (True/False)

(12) If A is solution produced using Runge-Kutta method of 2^{nd} order and B is solution produced using improved Euler method then B is more accurate than A. (True/False)

(13) Define Divided Differences.

(14) Which method can be used to find root of a polynomial?

(15) Minimum number of sub-intervals required to apply Simpson's 3/8 rule and Trapezoidal rule simultaneously is ________.

(16) Runge-Kutta method of 1^{st} order is similar as _____.

(17) Forward difference interpolation method is applicable for given domain data (Arguinerits) with equal length of subintervals. (True/False)

(18) If the value of $p = \dfrac{x - x_0}{h}$ lies in $\left[-\dfrac{1}{4}, 0\right]$, the best suitable interpolation to apply is ________.

(19) The iterative formula of Euler's method for solving $y' = f(x, y), y(x_0) = y_0$ is ________.

(20) Given y_0, y_1, y_2, y_3 Milne's Corrector formula to find y_4 for $y' = f(x, y)$ is ________.

2 (a) Attempt any **three** : **6**

(1) Define :

(i) Decision variable

(ii) Surplus variable.

(2) Write dual of Minimize $Z = 5x_1 + x_2 - 6x_3$

Subject to

$-2x_1 + x_2 - 11x_3 \le -2$

$-x_1 + 7x_2 + x_3 \ge 7$

$3x_1 - x_2 + 4x_3 \le 5$

and $x_1, x_2, x_3 \ge 0$.

(3) Explain canonical form of linear programming problem.

(4) Solve the following transportation problem using Lowest Cost Entry method.

		Destinations				Supply
		D_1	D_2	D_3	D_4	
Source	S_1	2	3	11	7	6
	S_2	1	0	6	1	1
	S_3	5	8	15	9	10
Demand		7	5	3	2	17

(5) Solve the following assignment problem.

		MEN		
		A	B	C
Task	1	120	100	80
	2	80	90	110
	3	110	140	120

(6) Write general mathematical form of an assignment problem.

(b) Attempt any **three** : **9**

(1) Define feasible solution and basic feasible solution. Also write differences between them.

(2) Solve the following assignment problem.

		MEN			
		A	*B*	*C*	*D*
Task	*I*	8	26	17	11
	II	13	28	4	26
	III	38	19	18	15
	IV	19	26	24	10

(3) Describe Vogel's approximation method.

(4) Explain primal-dual relationship for linear programming problem.

(5) Solve the following LPP using graphical method.

Minimize $Z = 3x_1 + 2x_2$

Subject to $5x_1 + x_2 \geq 10$

$x_1 + x_2 \geq 6$

$x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

(6) Solve the following LPP using simplex method.

Maximize $Z = 4x_1 + 3x_2$

Subject to condition

$2x_1 = x_2 \leq 1000$

$x_1 + x_2 \leq 800$

$x_1 \leq 400$

$x_2 \leq 700$

(c) Attempt any **two** : **10**

(1) Explain simplex algorithm.

(2) Solve the following assignment problem.

		Machines				
		M_1	M_2	M_3	M_4	M_5
Job	J_1	11	17	8	16	20
	J_2	9	7	12	6	15
	J_3	13	16	15	12	16
	J_4	21	24	17	28	26
	J_5	14	10	12	11	15

(3) Write algorithm of Hungarian method for assignment problem.

(4) Solve the following transportation problem using MODI method.

		Destinations				Supply
		D_1	D_2	D_3	D_4	
Source	S_1	21	16	25	13	11
	S_2	17	18	14	23	13
	S_3	32	27	18	41	19
Demand		6	10	12	15	43

(5) Solve the following linear programming problem by Big-M method.

Maximize $Z = 5x_1 + 3x_2$

Subject to condition

$$2x_1 + 4x_2 \leq 12$$

$$2x_1 + 2x_2 = 10$$

$$5x_1 + 2x_2 \geq 10$$

$$x_1, x_2 \geq 0$$

3 (a) Attempt any **three** : **6**

(1) Derive first order derivative of Newton's-forward interpolation formula.

(2) If $f(x) = x^{-1}$, show that $f(a, b, c, d) = \frac{-1}{abcd}$

(3) Find $\int_0^1 y\, dx$ using Simpson's $(1/3)^{rd}$ rule from the following table :

x	0	0.25	0.5	0.75	1
y	1.0000	0.9896	0.9589	0.9089	0.8415

(4) Write working rule of Runge's method.

(5) Write two differences between Gauss-Backward interpolation and Lagrange's interpolation.

(6) Derive Stirling's interpolation formula.

(b) Attempt any **three** : **9**

(1) Derive Newton's forward interpolation formula from divided difference interpolation formula.

(2) Using Picard's method obtain a solution upto the second approximation to the equation $y' = x + y$ such that $y(0) = 1$. Also find $y(0.1)$.

(3) Find y' at $x = 0.04$ from data below using Bessel's interpolation formula.

x	0.01	0.02	0.03	0.04	0.05	0.06
y	0.1023	0.1047	0.1071	0.1096	0.1122	0.1148

(4) Write working rule for Runge-Kutta method of 4^{th} order to solve simultaneous first order differential equation.

(5) If $y_0, y_1, \dots\dots\dots y_6$ are the consecutive terms of a series, then using Lagrange's interpolation formula prove that

$$y_3 = 0.05(y_0 + y_6) - 0.3(y_1 + y_5) + 0.75(y_2 + y_4)$$

(6) Derive Gauss backward interpolation formula.

(c) Attempt any **two** : **10**

(1) Using Newton's divided difference interpolation, find $f(x)$ and $f(8)$ from the given data.

x	0	1	3	5	6	9
$f(x)$	–18	0	0	–248	6	13104

(2) Derive Newton-Cote's quadrature formula.

(3) Derive Milne-Thomson predictor-corrector formula.

(4) Derive Laplace-Everett's interpolation formula.

(5) Find y and z at $x = 0.1$ using Taylor's series method, given that $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$, $y(0) = 2$ and $z(0) = 1$.